Определение. При фиксированном $\mu \in (0, \mu^0]$ назовем ЛССВСЗ (1) $F(\mu)\{x, y\}$-управляемой, если замыкание множества $F(\mu)\{X, Y\}(\mu)$, образованного при этом μ, совпадает с замыканием множества $\operatorname{Im} F(\mu)$.

Теорема. *Если выполнены условия:*

1) $\operatorname{rank} \tilde{P}(z) = n_1 + n_2$ *для некоторого комплексного* z;
2) $\operatorname{rank} \tilde{N}(\lambda, z) = n_1 + n_2 \quad \forall \lambda \in \mathbb{C} : e^{\lambda h} = z, rank \tilde{P}(z) < n_1 + n_2$;
3) $\operatorname{rank} \tilde{L}(\lambda) = n_1 + n_2 + c$ *для некоторого комплексного* λ;

то ЛССВСЗ (1) $\{x, y\}$*-управляема в пространстве* $\mathcal{M}^2_{n_1+n_2}$ *для всех достаточно малых* $\mu > 0$.

Доказательство основано на применении к ЛССВСЗ метода пространства состояний [1], ранговых условий F-управляемости систем с запаздыванием [2] и анализе зависимости этих условий от малого параметра, выполненном аналогично [3].

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THEOREM ON EXISTENCE OF A UNIQUE SOLUTION WITH GIVEN PROPERTIES TO RELAY SYSTEM

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The problem on existence of forced periodic solutions in control systems containing discontinuous hysteresis nonlinearity is concerned. We study the n-dimensional system of ordinary differential equations with relay nonlinearity and external continuous influence. Systems of the considered class can be used as systems of the electric drive constructed with semi-conductor diodes and intended to regulate rotation frequency of a rotor with an asynchronous electric motor. It is also possible to use these systems when processes in electric chains of control systems with a nonideal relay and elements from ferromagnetic materials are described. We offer an approach for choosing coefficients (including feedback coefficients as well) of the system such that there exist the forced harmonic or subharmonic oscillatory modes with the certain configuration in phase space. This approach is based on general problems of nonlinear system dynamics stated by V. I. Zubov, methods of canonical transformation theory, and the method of sections for system parameter space suggested for autonomous systems by R. A. Nelepin. To investigate the system, we use exact analytical methods, namely, the method of images and the fixed point method. We stress that the approach allows to find analytically switching instants and switching points of the image point of the required solution from an auxiliary system of transcendental equations. The auxiliary system constructing and conditions of its solvability are given in [1]. This work develops the results obtained in [1].

We consider the problem related to the existence of continuous periodic solutions with two switching points fixed in phase space and the period multiple to the period of the function describing external influence. A switching point of a solution to the system in phase space is said to be a state of the system such that the nonlinear function takes one of its threshold numbers and changes an output number, i.e. the switch occurs in a relay. The main result of this research work is the following theorem. The proof of the theorem is found in [2].

Theorem. *Let the following conditions hold:*

1) *the external influence of the system is a T-periodic function containing the sum of two sine functions and a constant;*

2) *the virtual points of stability in phase space of the system lie out of the non-single-valued zone of the function describing hysteresis nonlinearity;*

3) *the initial system is reduced to the special canonical form by the nonsingular transformation if the system is completely controllable with respect to the nonlinearity and the eigenvalues of the system matrix are real, prime, and nonzero;*

4) *the coefficients of the real vector defining feedback in the canonical system are nonzero except for one, which is zero;*

5) *the switching instants and the switching points of the image point of the solution to the canonical system are the solutions to the auxiliary system of transcendental equations constructing of which is based on the assumption that there exists at least one periodic solution with two switching points and which parameters satisfy the conditions of its solvability.*

Then, for given $k \in \mathbb{N}$, there exists a unique kT-periodic solution to the initial system with two switching points belonging to the switching hyperplanes, where the switching points can be calculated with the inverce transformation.

Remark. The second switching instant equals the period T provided the image point begins its motion at $t = 0$.

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POLE ASSIGNMENT IN DISCRETE-TIME LINEAR SYSTEMS WITH INCOMPLETE STATE FEEDBACK

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Consider a dicrete-time linear control system

$$x(t+1) = A(t)x(t) + B(t)u(t), \quad y(t) = C^*(t)x(t), \quad t \in \mathbb{Z}, \quad (x, u, y) \in \mathbb{K}^{n+m+k}, \qquad (1)$$

with a linear incomplete feedback $u(t) = U(t)y(t)$, where $\mathbb{K} = \mathbb{C}$ or $\mathbb{K} = \mathbb{R}$. By $X(t, s)$ denote the Cauchy matrix of unforced system $x(t+1) = A(t)x(t)$. System (1) is called *consistent on an interval* $[t_0, t_1)$ if for any $n \times n$-matrix $G \in M_n$ there exists a feedback gain $\widehat{U}(t)$, $t \in [t_0, t_1)$, that transfers the solution of the $n \times n$-matrix system

$$Z(t+1) = A(t)Z(t) + B(t)\widehat{U}(t)C^*(t)X(t, t_0), \quad t \in \mathbb{Z},$$